Math 10A
Worksheet, Discussion #34; Thursday, 8/2/2018
Instructor name: Roy Zhao

---

1 Eigenvalues and Eigenvectors

1.1 Concepts

1. An eigenvalue eigenvector pair for a square matrix A is a scalar λ and nonzero vector $\vec{v}$ such that $A\vec{v} = \lambda\vec{v}$. To find this, we write $\lambda\vec{v} = \lambda I\vec{v}$ and bring this to the other side to get $(A - \lambda I)\vec{v} = 0$. Since $\vec{v}$ is nonzero, this means that $(A - \lambda I)\vec{w} = 0$ has at least two solutions (since the trivial solution is a solution), and hence there must be an infinite number of solutions and $\det(A - \lambda I) = 0$.

   So to find the eigenvalues, we solve $\det(A - \lambda I) = 0$. For a particular eigenvalue, to find the associated eigenvector, we have to use Gaussian elimination on $A - \lambda I$ to get the general solution.

1.2 Problems

2. True False Associated to every eigenvalue is an eigenvector and vice versa

3. True False If 2 is an eigenvalue for A, then 4 is an eigenvalue for A^2.

4. True False If $\det(A) = 0$, then 0 has to be an eigenvalue of A.

5. True False If 2 is an eigenvalue of A and 3 is an eigenvalue of B, then $2 \cdot 3 = 6$ is an eigenvalue of AB.

6. True False For each eigenvalue, there is only one choice of eigenvector.

7. Find the eigenvalue and associated eigenvectors of $A = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$.

8. Find the eigenvalues and eigenvectors of $\begin{pmatrix} 1 & 3 \\ 9 & -5 \end{pmatrix}$.

9. Find the eigenvalues of $\begin{pmatrix} 2 & 4 & 4 \\ -1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix}$.

2 Linear Systems of Differential Equations

2.1 Concepts

10. In order to solve a system of linear differential equations, we represent it in the form $\vec{y}' = A\vec{y}$. Then we find the eigenvalues of A, say λ_1, λ_2. If $\lambda_1 \neq \lambda_2$ are real, then we find the eigenvectors $\vec{v_1}, \vec{v_2}$ and the general solution is of the form $\vec{y} = c_1 e^{\lambda_1 t}\vec{v_1} + c_2 e^{\lambda-2t}\vec{v_2}$.

2.2 Problems

11. True False If two matrices A, B have the same eigenvalues, then they have the same solutions to $\vec{y}' = A\vec{y}$.

12. Find the general solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = y_1(t) + 4y_2(t) \\ y_2'(t) = 3y_2(t) \end{cases}$$

13. Find the solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = 5y_1(t) - 4y_2(t) \\ y_2'(t) = 4y_1(t) - 5y_2(t) \end{cases}$$

with $\vec{y}(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

14. Verify that $\vec{x}(t) = \begin{pmatrix} 0 \\ -e^t \\ e^t \end{pmatrix}$, $\vec{y}(t) = \begin{pmatrix} e^{2t} \\ -2e^{2t} \\ 0 \end{pmatrix}$, $\vec{z}(t) = \begin{pmatrix} 0 \\ e^{3t} \\ e^{3t} \end{pmatrix}$ are solutions to $\vec{v}' = A\vec{v}$ where $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$.

15. Under the same notation as the previous problem. Write out the system of linear equations that $\vec{v}' = A\vec{v}$ represents and find the general solution.

16. Still with the same notation, what are the eigenvalues and eigenvectors of A?